# Vibration suppression of flexible space structures via trajectory optimization of manipulators in assembly tasks

Yuhang Liu<sup>1</sup>, Kai Luo<sup>2</sup>

 School of Aerospace Engineering Beijing Institute of Technology
 South Zhongguancun Street, 100081 Beijing, China liuyuhang1812@163.com  <sup>2</sup> School of Aerospace Engineering Beijing Institute of Technology
 5 South Zhongguancun Street, 100081 Beijing, China kailuo@bit.edu.cn

## **EXTENDED ABSTRACT**

#### 1 Introduction

The construction of large space structures, such as space power stations and satellite antennas, depends on space assembly technologies. For these structures, slender rods made of flexible materials can gain the advantages of reducing weight and volume. However, the motion of the manipulators will cause structural vibration, especially when they are operating rapidly. In order to suppress the residual vibration of flexible structures and improve the operating efficiency of manipulators, the trajectories of the space manipulators need to be optimized [1, 2]. Herein, we use the absolute nodal coordinate formulation (ANCF) method [3] to model the space structures including manipulators and flexible bases. To enforce the dynamic behavior of flexible multibody systems approaching that of rigid multibody systems and thus reduce the vibration, the trajectory of the manipulators are further optimized by the gradient descent method with sensitivity analysis [4]. The proposed method can describe complex space structures, improve the operation efficiency of complex space assembly tasks, and make the application of large flexible space structures possible.

#### 2 Methods

The dynamic equations of space mechanisms and structures with flexible components are established by using the ANCF method, and can be expressed as:

$$\mathbf{G} = \begin{bmatrix} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{\mathsf{T}} \boldsymbol{\lambda} - \mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}}) \\ \mathbf{\Phi}(\mathbf{q}, \mathbf{b}, t) \end{bmatrix} = \mathbf{0}$$
(1)

The angle driving constraint equation of the motors can be written as

$$\Phi(\mathbf{q}, \boldsymbol{\theta}(t)) = \mathbf{0} \tag{2}$$

and the corresponding design parameters of the joint are the motor angle values at discrete time points:

$$b_{i}^{k} = \theta^{k}(t_{i}), i = 0, 1, 2, \dots, p^{k}$$
(3)

where  $\mathbf{b}^k$  is the vector of the design parameter of joint number k, and  $\theta^k$  is the angle of this motor. In order to obtain C2continuity to prevent infinite motor torques, motor angles are discretized by piecewise cubic polynomial interpolation. The motor trajectory is optimized to reduce the position deviation caused by the vibration of the flexible link in the fast motion process as well as the residual vibration after the motion. After obtaining the ideal dynamic calculation results of the multirigid-body system through the target trajectory, our goal is to reduce the impact of the flexible components, and make the dynamic behavior of the rigid-flexible coupling system similar to that of the multi-rigid-body system, so we obtain the objective function as:

$$\Psi(\mathbf{q}(\mathbf{b}),\mathbf{b}) = \int_{t_0}^{t_1} F(\mathbf{q}(\mathbf{b},t)) dt = \int_{t_0}^{t_1} (\sum_{i=1}^R w_p^i \frac{1}{2} \|\mathbf{r}^i - \tilde{\mathbf{r}}^i\|^2 + \sum_{i=1}^R w_o^i \frac{1}{2} ((\mathbf{r}_x^i \cdot \tilde{\mathbf{r}}_z^i)^2 + (\mathbf{r}_y^i \cdot \tilde{\mathbf{r}}_z^i)^2 + (\mathbf{r}_z^i \cdot \tilde{\mathbf{r}}_x^i)^2)) dt$$
(4)

where  $\mathbf{r}_x^i$ ,  $\mathbf{r}_y^i$ ,  $\mathbf{r}_z^i$  and  $\tilde{\mathbf{r}}_x^i$ ,  $\tilde{\mathbf{r}}_y^i$ ,  $\tilde{\mathbf{r}}_z^i$  represent the actual and expected target slope vectors of *i*th rigid body component respectively, and  $w_p^i$  represents the weight of the *i*th rigid body orientation deviation. In order to make complex and flexible adjustments to the trajectory, we have a large number of discrete design variables, which can reach hundreds. At this time, intelligent algorithms like PSO method are inefficient, so we use the gradient descent method. Here we use the adjoint variable method to calculate the gradient, and the adjoint DAEs can be simplified according to the characteristics of the ANCF method into:

$$\mathbf{V} = \begin{bmatrix} \ddot{\boldsymbol{\mu}}^{\mathrm{T}} \mathbf{M} + \boldsymbol{\mu}^{\mathrm{T}} \mathbf{Q}_{q} + \boldsymbol{\mu}^{\mathrm{T}} \left( \left( \boldsymbol{\Phi}_{q}^{\mathrm{T}} \hat{\boldsymbol{\lambda}} \right)_{q} - \mathbf{Q}_{q} \right) + \boldsymbol{v}^{\mathrm{T}} \boldsymbol{\Phi}_{q} - F_{q} \\ \boldsymbol{\mu}^{\mathrm{T}} \boldsymbol{\Phi}_{q}^{\mathrm{T}} \end{bmatrix} = \mathbf{0}$$
(5)

where  $\mu$ ,  $\nu$  are adjoint variables, and the matrix **M** and  $\mathbf{Q}_{q}$  at each time step can be stored during the forward integration of the system dynamic equation (1). After the calculation of adjoint variables, the gradient  $\Psi_{b}$  can be simplified and calculated as:

$$\Psi_{\mathbf{b}} = \int_{t_0}^{t_1} \left( F_{\mathbf{b}} - \boldsymbol{\mu}^{\mathrm{T}} \left( \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}} \hat{\boldsymbol{\lambda}} \right)_{\mathbf{b}} - \boldsymbol{\nu}^{\mathrm{T}} \boldsymbol{\Phi}_{\mathbf{b}} \right) \mathrm{d}t$$
(6)

and gradient based nonlinear optimization methods can be used to optimize the design parameters **b**.

#### 3 Cases study

The deformation of slender beam made of flexible materials is beyond the range of linear vibration and cannot be accurately described by the assumed mode method. For the flexible link manipulator with a load at the end, we give a fast moving target trajectory, optimize the driving parameters to achieve high tracking accuracy, and significantly suppress the residual vibration of the flexible link (Fig. 1), which shows the effectiveness of the optimization method.



Figure 1: Optimized motion of a flexible link manipulator

Considering the more complex large space structure, we have established the model of two manipulators on a flexible base moving objects simultaneously (Fig. 2). The dynamic model can be easily established by using the ANCF method. The base vibration caused by the operation of one manipulator will affect the motion of the other manipulator. The simultaneous optimization of the motion of the two manipulators can suppress the residual vibration and improve the work efficiency while completing the task quickly.



Figure 2: Optimization effect of simultaneous operation of two manipulators on flexible base

### 4 Conclusion

In order to reduce the vibration of the flexible base caused by the rapid operation of the manipulators, the gradient descent method is used to optimize the joint trajectory of the manipulator. The gradients of the objective function with respect to the design variables are calculated by the adjoint variable method. The effectiveness of the method is verified by experiments and calculations, which can improve the operational efficiency of on-orbit assembly of complex space structures.

## References

- Abe A. Trajectory planning for residual vibration suppression of a two-link rigid-flexible manipulator considering large deformation[J]. Mechanism and Machine Theory, 2009, 44(9): 1627-1639.
- [2] Hoshyari S, Xu H, Knoop E, et al. Vibration-minimizing motion retargeting for robotic characters[J]. ACM Transactions on Graphics (TOG), 2019, 38(4): 1-14.
- [3] Shabana A A. ANCF reference node for multibody system analysis[J]. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, 2015, 229(1): 109-112.
- [4] Wang S, Tian Q, Hu H, et al. Sensitivity analysis of deployable flexible space structures with a large number of design parameters[J]. Nonlinear Dynamics, 2021, 105(3): 2055-2079.